BIRÛNÎ'S METHODS ON FINDING THE SOLAR PARAMETERS

To calculate the eccentricity and the longitude of the apogee of the orbit of the sun, Hipparchus used the intervals of the seasons. Consequently, he observed the tropics and the equinoxes by using several instruments. After him, Ptolemy followed the same way. [1]

In 16th Century, as it is known, the astronomers dwelt upon the dissatisfaction of this method because of the difficulty of the observations of the tropics. [2] Are there any astronomers who took this problem up before 16th Century? We find the answer of this question in *Al Qânun al Mas'ûdî* of Birûnî. Birûnî examines the sun in the sixth chapter of his book. He calculates the eccentricity and the longitude of the apogee of the orbit of the sun by using the old method. After this he says, "To use the beginning of the seasons requires the determination of the tropics to which we have pointed the difficulty of their observations. [3]" And he adds, "As the variations of declinations around the tropic, in one day, causes the difficulty of the correct determination of the beginning of the season, so, the ones who are partisants of the novelties turned to the observations of the points where the differences of the declinations are greater than the tropics. Though the differences of the declinations near the aquinoxes are the greatest, in the middle of the seasons they are greater than the tropics. According to their places the quadrant from the middle of the Water Bearer to the middle of the Bull is called eastern quadrant, the oppo-

[1] Ptolemy, The Almagest. Translated by R. Catesby Talioferro. The Great Books of the Western World. Vol. 16, pp. 93-94.

[2] N. Copernicus, On the Revolutions of the Heavenly Spheres. Translated by Charles Glenn Wallis. The Great Books of the Western World. Vol. 16. pp. 659 - 669; J. Dreyer, Tycho Brahe. Edinburgh 1890, P. 248; Taqî al Dîn, Sıdra al Muntahâ. Kandilli Observatory 53a.

[3] Al-Qânûnu'l-Mas'ûdî (Canon Masudicus) Vol. II (An Encyclopaedia of Astronomical Sciences) Edited by the Bureau from the oldest extant Mss. Under the auspices of the Ministry of Education, Government of India. Hyderabad-Dn 1955, P. 656.

ان العمل بمبادى الفصول تضطر الى وقت الانقلاب الذى قد منا عسر الوقوف عليه

site of it western and from the middle of the Bull to the middle of the Lion northern, and the opposite of it southern."[4]

What does he mean by these? According to the places of the observations, the two lines joinning the tropics and equinoxes are perpendicular. Also the lines joinning the middle points of the seasons are perpendicular. So instead of using tropics and equinoxes the middle points of seasons can be used in determination of the orbit. He calculates the parameters as follows by using the afore mentioned quadrants.[5]

A represents the middle point of the Bull.

B the middle point of the Lion.

C the middle point of the Scorpion (Figure I).[6]

The arcs AB, BC, CA are measured by the observations. Consequently chords AB, BC and CA are given. HT, eccentricity, and the direction of HT, longitude of the apogee, are wanted. If we calculate the two sides of the right triangle HTZ, the hypotenuse HT is obtained.

$$\frac{AC}{2} - CT = TZ$$

$$CT = ?$$

$$\frac{\overline{AC}^2 + \overline{BC}^2 - \overline{AB}^2}{2\,AC} = CT$$

To find the place of the observation a perpendicular is drown to AC from B.T is the place of the observation.

[4] Al-Qânûnu'l-Mas'ûdî. P. 657.

ولما كان فى تحصيل المنقلب مافيه من العسير لكنه تفاضل الميل حوله فى اليوم الواحد على خلاف حاله عند الاعتدالين عدل المحدثون عن نقط التغير فى مبادى الفصول الى نقط تفاضل الميل فيها اكثر من تفاضله عند المنقلب وان كان أقل من تفاضله عند الاعتدال و تلك انصاف الفصول اعنى اوساط البروج الثوابت وليسم لحكاية وجودهم الربع الذى من نصف برج الدلو الى نصف برج الثور ربعا شرقياً ونظيره الذى يقابله غربياً و الذى من نصف برج الثور الى نصف البرج الاسد شمالياً و نظيره جنوبياً .

[5] Al-Qânûnu'l-Mas'ûdî. pp. 678-81.

[6] Al-Qânûnu'l-Mas'ûdî· Figure 100.

Birûnî wants to use sines instead of chords. For this reason he drows a small circle BKD which its diameter is equal to the half of the diameter of the first.

Now $FS = ?$

$$IM . IF = \overline{IK}^2$$

$$IK = FS$$

$$\sqrt{\overline{FS}^2 + \overline{SU}^2} = UF = \text{eccentricity}$$

After this Birûnî asks a question.

"Question: Could these two wanted quantities be obtained by another method?

Answer: If two points of observations are in opposition in ecliptic and the third is any desired place except being in the middle of the first they cause us to reach the necessary conclution."[7]

Birûnî makes the observations of three points. A and C of them are in opposition and B is any desired place.[8] Arcs AB, BC and CA are given. HT, the eccentricity and the direction of HT, the longitude of apogee are required (Figure II).[9]

In triangle BCT Angle $BCT = 1/2$ Arc AB

The angle BCT is given by means of the observation. The side BC is given. So the triangle is given. Finally CT is calculated.

$CZ - CT = TZ =$ one side of the triangle.

HT, the eccentricity is obtained from them. The angle HTZ gives the direction of HT, the longitude of apogee. This method is almost the same which was used by Copernicus, Taqî al Dîn and Tycho Brahe in 16th Century.[10]

Birûnî gives another example in which the points are selected without attaching any condition.[11]

A,B,C are observed points. The place of the observation is T (Figure III).[12]

[7] Al-Qânûnu'l-Mas'ûdî. P. 681.
[8] Al-Qânûnu'l-Mas'ûdî. pp. 681-82.
[9] Al-Qânûnu'l-Mas'ûdî. Figure 101.
[10] Note 2.
[11] Al-Qânûnu'l-Mas'ûdî. pp. 682-84.
[12] Al-Qânûnu'l-Mas'ûdî. Figure 102.

B and T are jointed and extended. It cuts the circle at D.

In triangle ATD, Angle ABD = 1/2 Arc AB

Angle ATD = 180°- Angle ATB

Angle ATB is given by the observation. Let TD be equal to I.

$$TD/DA = \text{Sin } \widehat{TAD}/\text{Sin}\widehat{ATD}$$

From this DA is known.

In triangle CTD, Angle TDC = 1/2BC

Angle CTD = 180° - Angle BTC

Angle BTC is given by the observation. Again let TD be equal to I.

$$TD/DC = \text{Sin}\widehat{TDC}/\text{Sin}\widehat{DTC}$$

DC can be calculated. So the chords AD and DC are calculated as DT being aqual to I.

Now, let the perpendicular CZ be drown to DA from C.

In the triangle CZD, Angle ZDC = 1/2 Arc AB + 1/2 Arc BC.

Let DC be aqual to I. So DZ will be equal to the sine of the angle C. Whereas CD is known TD being equal to I, so DZ can be obtained as TD being aqual to I.

$$\overline{DZ}^2 + \overline{CZ}^2 = \overline{CD}^2$$

$$DZ = \sqrt{\overline{CD}^2 - \overline{CZ}^2}$$

$$DA - DZ = AZ$$

$$AC = \sqrt{\overline{AZ}^2 + \overline{CZ}^2}$$

The amount of AC can be obtained as TD is to I.

On the other hand, Chord AC = Chord AB + Chord BC

This is calculated as the diameter being 120. Now, we can get the value of TD as the diameter of the circle being 120. So the chords AD and CD are calculated.

Now, let a perpendicular be drown from H to BD.

$$HM = \frac{\text{Arc AB} + \text{Arc AD} - \text{Kiriş } 180°}{2}$$

$$TM = \text{Kiriş } 60° - TD$$

$$\overline{HT}^2 = \overline{TM}^2 + \overline{HM}^2$$
$$HT = \sqrt{\overline{TM}^2 + \overline{HM}^2} = \text{The eccentricity.}$$

Birûnî adds another interesting method. Before explaining this let us dwell upon a problem which is not clearly solved in Islam. Did Muslims try to determine the orbits of the heavenly bodies by making daily observations through the whole year? Or were they satisfied with the observations of certain important points. In this matter Aydın Sayılı gives some decumans to us and shows that there are some records about the dailly observations in Islam. [13] For example regular daily observations of the sun and the moon were made for a whole year at the Qâsîyûn Observatory of Al Mamûn. [14] Jabir ibn Aflaḥ also speaks of daily observations of the sun [15]. According to Birjândî the intervals of the determinations of the planetary longitudes must be a single day. [16] Qutb al Dîn al Shîrâzî suggests that the movements of apogees based upon repeated observations made in Maraghâ Observatory. [17]

Did the Muslims determine the parameters of the orbits of the heavenly bodies using a new method depending these daily observations? There was not any record about this. Nevertless Birûnî's last example is a clear sign of this matter. He uses the daily observations of the sun through the whole year to determine the parameters of the sun.

He says as follows, "If a person makes the daily observations of the sun on the meridian through the whole year and chooses the equal arcs which was cut by the sun in equal times on the ecliptic, apogee is found between these two equal arcs." [18]

[13] The Observatory in Islam. Publications of the Turkish Historical Society, Series VII, No. 38. Ankara, 1960, P. 316.

[14] The Observatory in Islam. P. 316, S. Tekeli, *Nasirüddin, Takiyüddin ve Tycho Brahe'nin Rasat Aletlerinin Mukayesesi. Ankara Üniversitesi, Dil ve Tarih-Coğrafya Fakültesi Dergisi*, Vol. 16, No. 3-4, 1958, P. 388, 391

[16] The Observatory in Islam. P. 317.

[17] The Observatory in Islam. P. 320.

[18] Al-Qânûnu'l-Mas'ûdî. P. 684.
ايضا فن حصل له مواضع الشمس لنصف نهار كل يوم طول السنة ثم طلب قوسين من فلك البروج متساويتين كان الاوج متوسطا بينهما.

Here is the method.[19] If the arcs KA and AB or arcs ZA and AC are equal the apogee is found between them. As the triangle THF is known so the eccentricity and its direction are known (Figure III).[20]

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[19] Al-Qânûnu'l-Mas'ûdî. P. 684.

[20] Al-Qânûnu'l-Mas'ûdî. Figure 103.